CHAPTER 1

Sets and Probability

In a survey of 200 people that had just returned from a trip to Europe, the following information was gathered.

- 142 visited England
- 95 visited Italy
- 65 visited Germany
- 70 visited both England and Italy
- 50 visited both England and Germany
- 30 visited both Italy and Germany
- 20 visited all three of these countries

How many went to England but not Italy or Germany?

We will learn how to solve puzzles like this in the second section of the chapter when counting the elements in a set is discussed.

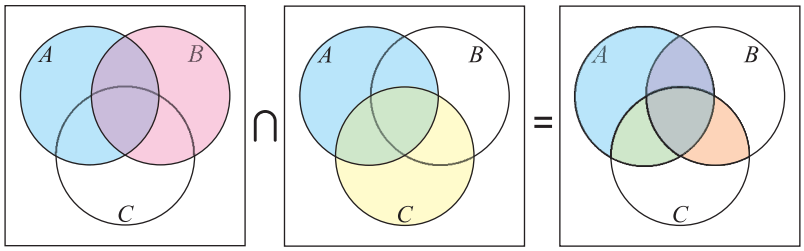

Figure 1.7b

We can summarize the laws we have found in the following list.

Laws for Set Operations

$A \cup B = B \cup A$	Commutative law for union
$A \cap B = B \cap A$	Commutative law for intersection
$A \cup (B \cup C) = (A \cup B) \cup C$	Associative law for union
$A \cap (B \cap C) = (A \cap B) \cap C$	Associative law for intersection
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive law for union
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive law for intersection
$(A \cup B)^c = A^c \cap B^c$	De Morgan law
$(A \cap B)^c = A^c \cup B^c$	De Morgan law

✧ Applications

EXAMPLE 7 **Using Set Operations to Write Expressions** Let U be the universal set consisting of the set of all students taking classes at the University of Hawaii and

$B = \{x | x \text{ is currently taking a business course}\}$
$E = \{x | x \text{ is currently taking an English course}\}$
$M = \{x | x \text{ is currently taking a math course}\}$

Write an expression using set operations and show the region on a Venn diagram for each of the following:

a. The set of students at the University of Hawaii taking a course in at least one of the above three fields.

b. The set of all students at the University of Hawaii taking both an English course and a math course but not a business course.

c. The set of all students at the University of Hawaii taking a course in exactly one of the three fields above.

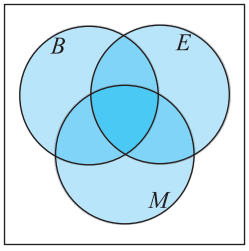

Figure 1.8a

Solution

a. This is $B \cup E \cup M$. See Figure 1.8a.

b. This can be described as the set of students taking an English course (E) and also (intersection) a math course (M) and also (intersection) not a business course (B^c) or

$$E \cap M \cap B^c$$

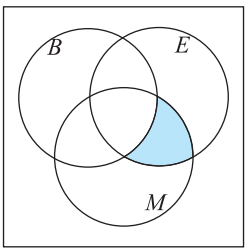

Figure 1.8b

This is the set of points in the universal set that are in both E and M but not in B and is shown in Figure 1.8b.

c. We describe this set as the set of students taking business but not taking English or math ($B \cap E^c \cap M^c$) together with (union) the set of students taking English but not business or math ($E \cap B^c \cap M^c$) together with (union) the set of students taking math but not business or English ($M \cap B^c \cap E^c$) or

$$(B \cap E^c \cap M^c) \cup (B^c \cap E \cap M^c) \cup (B^c \cap E^c \cap M)$$

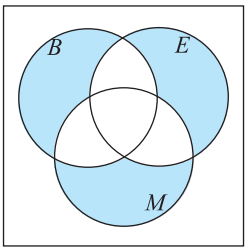

Figure 1.8c

This is the union of the three sets shown in Figure 1.8c. The first, $B \cap E^c \cap M^c$, consists of those points in B that are outside E and also outside M. The second set $E \cap B^c \cap M^c$ consists of those points in E that are outside B and M. The third set $M \cap B^c \cap E^c$ is the set of points in M that are outside B and E. The union of these three sets is then shown on the right in Figure 1.8c. ✦

REMARK: The word **only** means the same as exactly one. So a student taking only a business course would be written as $B \cap E^c \cap M^c$.

Self-Help Exercises 1.1

1. Let $U = \{1,2,3,4,5,6,7\}$, $A = \{l,2,3,4\}$, $B = \{3,4,5\}$, $C = \{2,3,4,5,6\}$. Find the following:

a. $A \cup B$ **b.** $A \cap B$
c. A^c **d.** $(A \cup B) \cap C$
e. $(A \cap B) \cup C$ **f.** $A^c \cup B \cup C$

2. Let U denote the set of all corporations in this country and P those that made profits during the last year, D those that paid a dividend during the last year, and L those that increased their labor force during the last year. Describe the following using the three sets P, D, L, and set operations. Show the regions in a Venn diagram.

a. Corporations in this country that had profits and also paid a dividend last year
b. Corporations in this country that either had profits or paid a dividend last year
c. Corporations in this country that did not have profits last year
d. Corporations in this country that had profits, paid a dividend, and did not increase their labor force last year
e. Corporations in this country that had profits or paid a dividend, and did not increase their labor force last year

1.1 Exercises

In Exercises 1 through 4, determine whether the statements are true or false.

1. **a.** $\emptyset \in A$ **b.** $A \in A$

2. **a.** $0 = \emptyset$ **b.** $\{x,y\} \in \{x,y,z\}$

3. **a.** $\{x|0 < x < -1\} = \emptyset$
b. $\{x|0 < x < -1\} = 0$

4. **a.** $\{x|x(x-1) = 0\} = \{0,1\}$
b. $\{x|x^2 + 1 < 0\} = \emptyset$

5. If $A = \{u,v,y,z\}$, determine whether the following statements are true or false.

a. $w \in A$ **b.** $x \notin A$
c. $\{u,x\} \cup A$ **d.** $\{y,z,v,u\} = A$