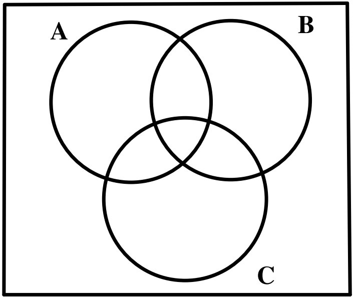
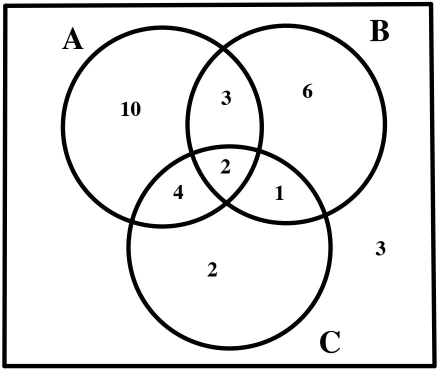
Venn Diagrams of 3 sets

A Venn diagram of 3 sets divides the universal set into 8 non-overlapping regions. We can sometimes use partial information about numbers in some of the regions to derive information about numbers in other regions or other sets.



Venn Diagrams of 3 sets

Example The following Venn diagram shows the number of elements in each region for the sets A, B and C which are subsets of the universal set U.



Find the number of elements in each of the following sets:

(a) $A \cap B \cap C$ 2

(b) B' $3 + 2 + 4 + 10 = 19$

(c) $A \cap B$ $3 + 2 = 5$

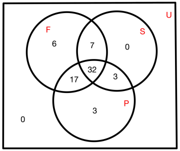
(d) C $2 + 4 + 2 + 1 = 9$

(e) $B \cup C$ $9 + 3 + 6 = 18$

Venn Diagrams of 3 sets

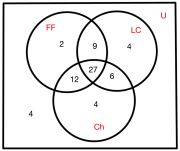
Example In a survey of a group of 68 Finite Math students, 62 liked the movie "The Fault in our Stars", 42 liked the movie "The Spectacular Now" and 55 liked the movie "The Perks of Being a Wallflower". 32 of them liked all 3 movies, 39 of them liked both "The Fault in Our Stars" and "The Spectacular Now", 35 of them liked both "The Spectacular Now" and "The Perks of Being a Wallflower" and 49 of them liked both "The Fault in Our Stars" and "The Perks of Being a Wallflower". Represent this information on a Venn Diagram. $n(U) = 68;$

$n(F) = 62;\ n(S) = 42;\ n(P) = 55.\ n(F \cap S) = 39;$
$n(P \cap S) = 35;\ n(F \cap P) = 49.\ n(F \cap S \cap P) = 32.$



Venn Diagrams of 3 sets

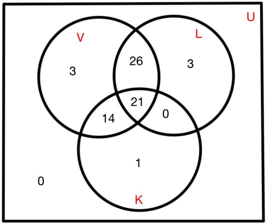
Example In a survey of a group of 68 Finite Math students (Spring 2006), 50 said they liked Frosted Flakes, 49 said they liked Cheerios and 46 said they liked Lucky Charms. 27 said they liked all three, 39 said they liked Frosted Flakes and Cheerios, 33 said they liked Cheerios and Lucky Charms and 36 said they liked Frosted Flakes and Lucky Charms. Represent this information on a Venn Diagram. How many didn't like any of the cereals mentioned? $n(U) = 68$; $n(FF) = 50$; $n(Ch) = 49$; $n(LC) = 46$. $n(FF \cap Ch) = 39$; $n(LC \cap Ch) = 33$; $n(FF \cap LC) = 36$. $n(FF \cap Ch \cap LC) = 27$.



Venn Diagrams of 3 sets

Example The results of a survey of 68 Finite Math students(Spring 2006) on learning preferences were as follows: 64 liked to learn visually, 50 liked learning through listening and 36 liked learning Kinesthetically. 21 liked using all three channels, 47 liked to learn visually and through listening, 35 liked to learn both visually and kinesthetically, 21 liked to learn through listening and kinesthetically. How many preferred only visual learning?

$n(U) = 68$; $n(V) = 64$; $n(L) = 50$; $n(K) = 36$;
$n(V \cap L) = 47$; $n(V \cap K) = 35$; $n(L \cap K) = 21$.
$n(V \cap L \cap K) = 21$.

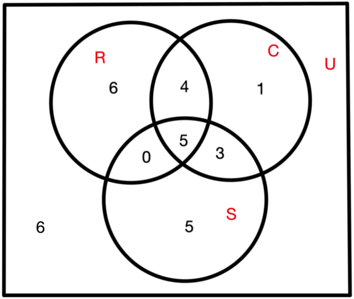


Old Exam questions for Review

1 In a group of 30 people, 15 run, 13 swim, 13 cycle, 5 run and swim, 8 cycle and swim, 9 run and cycle, and 5 do all three activities. How many of the 30 people neither run nor cycle?

(a) 8 (b) 10 (c) 9 (d) 12 (e) 11

$n(U) = 30$; $n(R) = 15$; $n(S) = 13$; $n(C) = 13$.
$n(R \cap S) = 5$; $n(C \cap S) = 8$; $n(R \cap C) = 9$.
$R \cap C \cap S = 5$.



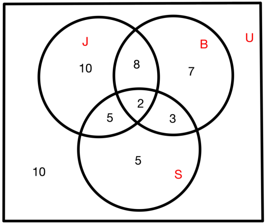
Answer is $6 + 5 = 11$ or (e)

Old Exam questions for Review

2 Out of 50 students who exercise regularly, 25 jog, 20 play basketball and 15 swim. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 2 people do all three. How many students do not do any of these activities?

(a) 10 (b) 15 (c) 4 (d) 0 (e) 2

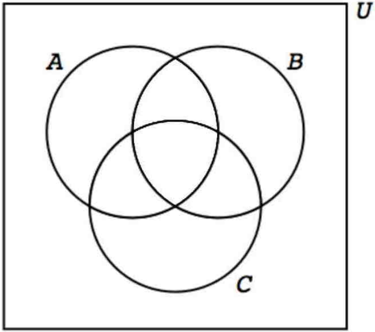
$n(U) = 50$; $n(J) = 25$; $n(B) = 20$; $n(S) = 15$.
$n(B \cap J) = 10$; $n(B \cap S) = 5$; $n(J \cap S) = 7$;
$n(B \cap J \cap S) = 2$;



Answer is 10 or (a)

Worked homework problem

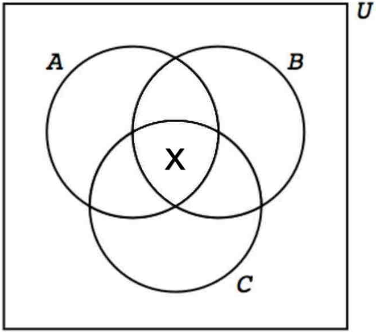
Here is an example of a type of problem from the homework. Given 3 subsets A, B and C of a universal set U, suppose $n(U) = 68$; $n(A \cup B \cup C) = 64$; $n(A) = 50$; $n(B) = 49$; $n(C) = 46$. $n(A \cap B) = 39$; $n(C \cap B) = 33$; $n(A \cap C) = 36$. Fill in the Venn diagram.



We do not know $n(A \cap B \cap C)$ or this would just be another example of earlier problems.

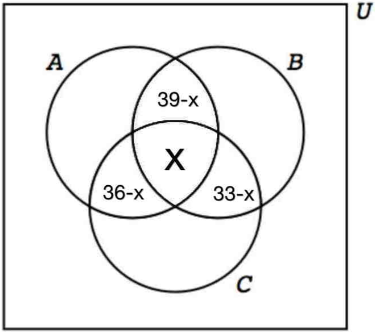
Worked homework problem

Denote $n\left(A \cap B \cap C\right)$ by x.



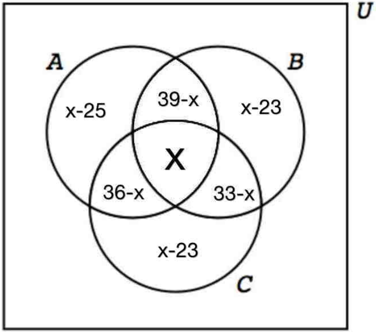
Worked homework problem

Now work out the double intersections: $n\left(A \cap B\right) = 39$; $n\left(C \cap B\right) = 33$; $n\left(A \cap C\right) = 36$.



Worked homework problem

Now work out the sets: $n(A) = 50$; $n(B) = 49$; $n(C) = 46$.



For example, if y denotes the part of A outside of $B \cup C$,

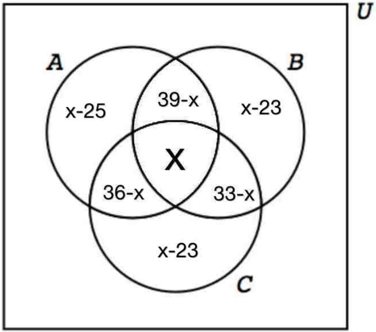
$$50 = y + (39 - x) + (36 - x) + x = y + 75 - x$$

so

$$y = x - 25$$

The others are similar.

Worked homework problem



Since

$$64 = n(A \cup B \cup C) = x + (39 - x) + (36 - x) + (33 - x) \\ + (x - 25) + (x - 23) + (x - 23) = x + (108 - 71) = \\ x + 37$$

Hence $n((A \cup B \cup C)^c) = 68 - 64 = 4$.

Worked homework problem